Appendix A

A. Gold Sequence Generator

Due to the large range of values of the cross-correlations of the maximum-length shift-register sequence, sometimes called *m*-sequence, they usually are not adopted to CDMA-like applications. PN-sequences with better periodic cross-correlation properties than *m*-sequences are given by the sequences of Gold (1967, 1968) and Kasami (1966), where the two-values of their sequences are +1 and −1. In Robert Gold, "Maximal recursive sequences with 3-valued recursive cross-correlation functions," *IEEE Trans. on Inform. Theory*, vol. 41, pp. 154-156, January 1968 [21], the contents of which are hereby expressly incorporated by reference, it was proved that certain pairs of *m*-sequences of length N exhibit a three-valued cross-correlation function [21] with the values -1, $-t(m)$, $t(m)-2\}$, where

$$t(m) = \begin{cases} 2^{(m+2)/2}+1 & \text{(odd } m) \\ 2^{(m+2)/2}+1 & \text{(even } m) \end{cases}$$

and where m denotes an m-stage shift register of sequence length $N = 2^m - 1$.

Consider the generation of the Gold sequences of length $N = 31 = 2^5 - 1$ with the pair of preferred sequences, that are obtained from the book by Peterson and Weldon (W. W. Peterson and E. J. Weldon, *Error Correcting Codes*, MIT Press, Cambridge, MA, 1972 [22], the contents of which are hereby expressly incorporated by reference) are described by the polynomials,

$$\begin{aligned} g_1(D) &= D^5+D^2+1, \\ g_2(D) &= D^5+D^4+D^3+D^2+1 \end{aligned}$$

Let b and b' represent the two m-sequences with period $N = 31$ that are generated by $g_1(D)$ and $g_2(D)$, respectively. The Gold family for the sequences of length 31 comprises of $2^m+1 = 33$ sequences given in R. L. Peterson, R. E. Ziemer, and D. E. Borth, *Introduction to Spread Spectrum Communications*, Prentice Hall, Englewood Cliffs, NJ, 1995 [23], the contents of which are hereby expressly incorporated by reference, by

$$G = \{b\} \cup \{b'\} \cup \{\{b + D^{\tau}b'\} \mid 0 \leq \tau \leq N-1\}, \quad (56)$$

where the term $D^{\tau}b'$ represents a phase shift of the m-sequence b' by τ units and "U" denotes set union. The sequences s_1 and s_2 in the example of Section 2 are derived by setting τ in Eq. (56) equal to 0 and 1, respectively.